Problem of the week

Nuclear Physics

(a) Outline how the results of Rutherford-Geiger-Marsden experiment led to the nuclear model of the atom.

(b) The binding energy per nucleon for the nuclide $^{137}_{58}Ce$ is 8.391827 MeV.

(i) State what the binding energy per nucleon is a measure of.

Calculate the mass of the nucleus of $^{137}_{58}Ce$ in

(ii) u,

(iii) MeV c^{-2},

(iv) kg.

(c) Calculate the energy released in the beta plus decay $^{23}_{12}Mg \rightarrow {}^{23}_{11}Na + e^{+} + \nu$. The **atomic** masses are:

Mg 22.994124 u

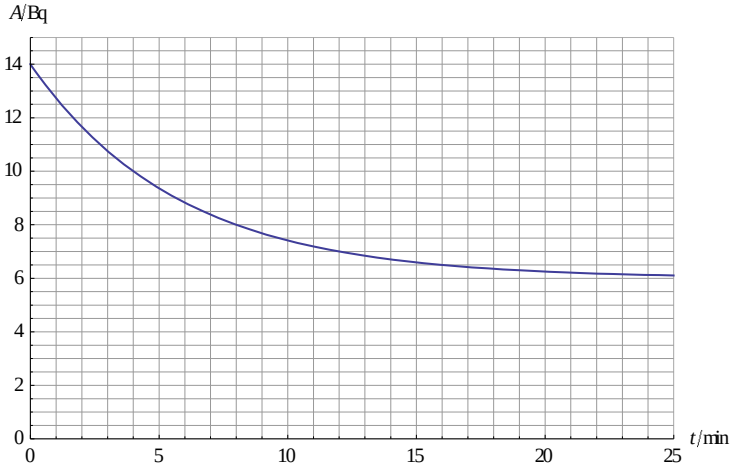
Na 22.989769 u

(d) Outline what is meant by the statement that radioactive decay is random and spontaneous.

(e)

(i) A radioactive element X decays into a stable element Y. Calculate the ratio of the number of Y atoms to the number of X atoms after 3 half-lives have gone by. No Y atoms were present initially.

(ii) The graph shows the variation with time of the activity of X.



Determine the half-life of X.

Answers

(a) Very few of the alpha particles were deflected by very large angles. This could be explained if the entire positive charge of the atom was concentrated in a very small volume, the nucleus.

(b)

(i) It is a measure of how tightly bound the nucleus is.

(ii) The binding energy is 8.391827×137 = 1149.68 MeV. So, the mass defect is

$$\mu = \frac{1149.68}{931.5} = 1.23422 \text{ u}.$$

But $\mu = 58m_p + 79m_n - M \Rightarrow M = 58m_p + 79m_n - \mu = 136.8 \text{ u}$.

(iii) $M = 136.8 \times 931.5 = 1.27 \times 10^5$ MeV c^{-2}.

(iv) $M = 136.8 \times 1.66 \times 10^{-27} = 2.27 \times 10^{-25}$ kg .

(c) $Q = \Delta mc^2 = (M_{Mg} - M_{Na} - m_e)c^2$, where the masses are nuclear masses. In terms of atomic masses $M_{Mg} = 22.994124 - 12m_e$ and $M_{Na} = 22.989769 - 11m_e$. Hence

$$\Delta m = (22.994124 - 12m_e) - (22.989769 - 11m_e) - m_e$$

$$Q = (22.994124 - 22.989769 - 2m_e)c^2$$

$$Q = 3.25784 \times 10^{-3} \times 931.5 = 3.03 \text{ MeV}$$

(d) Random: it cannot be predicted which nucleus will decay and when. Spontaneous: the decay cannot be affected or controlled in any way.

(e)

(i) If the initial number of x atoms was N, after 3 half-lives the number of X atoms has been reduced to $\frac{N}{8}$ and so the number of Y atoms is $\frac{7N}{8}$. The ratio is then 7.

(ii) There is a background count rate of 6 Bq. Hence the initial activity is 8 Bq. After a half-life the activity halves to 4 Bq and with the background the count rate would be 10 Bq. From the graph this corresponds to 4 min.